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Analytic solution to the conjugate heat transfer problem of flow past a heated block

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Abstract—An analytic solution has been developed for steady laminar two-dimensional conjugated heat transfer across a rectangular block with constant internal heat source and subjected to forced convection on top, different thermal boundary conditions at the sides, and adiabatic wall condition at the bottom. The idealized problem solution is of interest in the basic analysis of single-block cooling. Numerical results presented illustrate the effects of the Peclet number, conductivity ratio and type of thermal boundary condition on the temperature profiles and heat flux distributions along the upper surface of the block.

1. INTRODUCTION

THE PROBLEM of heat transfer from a heated block subjected to convective cooling at its upper surface is of interest in numerous engineering applications including electronic equipment cooling, heat exchangers, material processing and thermal building assessment. A number of relevant papers employing analytic or approximate methods have been published in recent years [1–7]. Luikov [1] considered heating of a thin flat plate of semi-infinite extent by forced convection and solved the conjugated problem analytically assuming constant velocities in the momentum boundary layer and a linear temperature profile for the thin plate. Karvinen [2] used the integral method to solve a problem similar to that considered by Luikov and achieved good agreement with measured data. Sparrow *et al.* [3] examined experimentally fluid flow and heat transfer parameters for several heated blocks (flat packs) in a cooling channel. Braaten and Patankar [4] as well as Incropera *et al.* [5] considered a shrouded array of blocks using numerical solution methods. Davalath and Bayazitoglu [6] analyzed numerically conjugate heat transfer for two-dimensional, developing flow over several heated blocks. Rizk and Kleinstreuer [7] studied numerically forced convection cooling of a linear array of heated blocks in open and porous material-filled channels.

This paper considers steady laminar boundary-layer type flow over the top surface of a block with a constant volumetric heat source. The coupling between the block and the fluid is through the unknown block surface temperature which varies in the axial direction. The boundary conditions considered at different surfaces of the block include: a constant temperature for the inlet side and isothermal surface or constant wall heat flux for the exit side. Of practical interest are the location and magnitude of the highest temperature in the block, the temperature rise of the coolant, the average heat transfer coefficient and the thermal penetration depth.

2. MATHEMATICAL FORMULATION

Conjugated steady conduction–convection heat transfer from a rectangular block with volumetric heat source to a free stream as illustrated in Fig. 1 is considered. The temperature fields in these two regions are coupled via compatibility conditions at the wall–fluid interface which include the continuity of temperature and heat flux at the interface. The axial and normal velocity components, u and v, are assumed to be constant, with $|v/u| \ll 1$. Thus, the model represents thermal boundary-layer type flow past the surface of the heated block with a (small) mass flux normal to the block surface. The mathematical formulation of the problem for conduction in the block, forced convection flow heat transfer and the interfacial conditions are now given.

2.1. *Conduction inside block*

The steady two-dimensional temperature field ψ in the block with energy source is governed by

$$\frac{\partial^2\psi}{\partial\hat{x}^2}+\frac{\partial^2\psi}{\partial\hat{y}^2}+\frac{g}{k_s}=0,\quad 0<\hat{x}<L,\quad -l<\hat{y}<0 \tag{1a}$$

subject to the boundary conditions

$$\psi = T_0 \quad \text{at } \hat{x}=0,\quad -l<\hat{y}<0 \tag{1b}$$

$$\frac{\partial\psi}{\partial\hat{x}}=0 \quad \text{or} \quad \psi=T_0 \quad \text{at } \hat{x}=L,\quad -l\leqslant\hat{y}\leqslant 0 \tag{1c}$$

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NOMENCLATURE

A	aspect ratio, l/L
$f(x)$	nondimensional wall–fluid interface temperature
g	heat generation rate [W m^{-3}]
k_s	solid thermal conductivity [W m^{-1} K^{-1}]
k_f	fluid thermal conductivity [W m^{-1} K^{-1}]
k^*	dimensionless conductivity ratio, k_s/k_f
l	block height [m]
L	block length [m]
N_m	normalization integral
Nu	Nusselt number
Pe	Peclet number, uL/α
$\hat{q}_w(\hat{x})$	dimensional upper block heat flow rate [W]
$q_w(x)$	nondimensional heat flux at the interface, $\hat{q}k_s/lgk_f$
t	dimensionless fluid temperature, $(T-T_0)/(lLg/k_s)$
T_∞	ambient temperature [K]
T	fluid temperature [K]
u, v	horizontal and vertical velocity components [m s^{-1}]
$\hat{x}, \hat{y}$	dimensional coordinates [m]
x, y, y^*	dimensionless coordinates
$X_m(x)$	eigenfunctions.

Greek symbols

α	thermal diffusivity [m^2 s^{-1}]
β_m	eigenvalues
γ	velocity ratio, $\|u/v\|$
$\hat{\delta}_{th}$	dimensional penetration depth [m]
δ_{th}	dimensionless penetration depth, $\hat{\delta}_{th}/L$
δ_{ij}	Kronecker delta
θ	dimensionless block temperature, $(\psi-T_0)/(lLg/k_s)$
ψ	block temperature [K].

and

$$\frac{\partial \psi}{\partial \hat{y}} = 0 \quad \text{at} \quad \hat{y} = -l, \quad 0 \leqslant \hat{x} \leqslant L. \tag{1d}$$

The boundary condition at the block–fluid interface along $\hat{y} = 0$ is given in Section 2.3.

Introducing the following dimensionless variables:

$$\theta = (\psi - T_0)/(lLg/k_s), \quad x = \hat{x}/L, \quad y^* = -\hat{y}/L \quad \text{and} \quad A = \frac{l}{L} \tag{2a–d}$$

the system (1) takes the form

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^{*2}} + A^{-1} = 0, \quad 0 < x < 1, \quad 0 < y^* < A \tag{3a}$$

where

$$\theta = 0 \quad \text{at} \quad x = 0 \tag{3b}$$

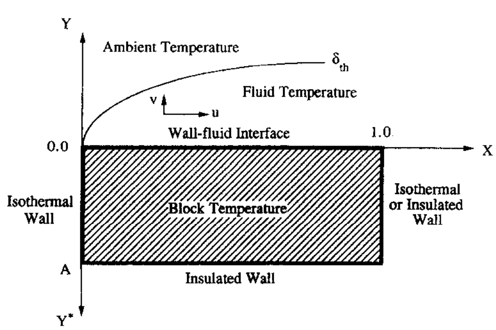

FIG. 1. System sketch with coordinates.

$$\frac{\partial \theta}{\partial x} = 0 \quad \text{or} \quad \theta = 0 \quad \text{at} \quad x = 1 \tag{3c}$$

and

$$\frac{\partial \theta}{\partial y^*} = 0 \quad \text{at} \quad y^* = A. \tag{3d}$$

2.2. *Forced convection over block surface*

The steady two-dimensional energy equation for forced convection of a constant property fluid is taken as

$$u\frac{\partial T}{\partial \hat{x}} + v\frac{\partial T}{\partial \hat{y}} = \alpha \frac{\partial^2 T}{\partial \hat{y}^2}, \quad \hat{x} > 0, \quad \hat{y} > 0 \tag{4a}$$

subject to the boundary conditions

$$T = T_0 \quad \text{at} \quad \hat{x} = 0 \tag{4b}$$

and

$$T \to T_0 \quad \text{as} \quad \hat{y} \to \infty. \tag{4c}$$

The condition at the solid surface along $\hat{y} = 0$ is given in Section 2.3.

This forced convection problem (4) can be expressed in dimensionless form as

$$\frac{\partial t}{\partial x} + \gamma \frac{\partial t}{\partial y} = \frac{1}{Pe}\frac{\partial^2 t}{\partial y^2}, \quad x > 0, \quad y > 0 \tag{5a}$$

$$t = 0 \quad \text{at} \quad x = 0 \tag{5b}$$

$$t \to 0 \quad \text{as} \quad y \to \infty \tag{5c}$$

where the dimensionless quantities are defined as

$$t = \frac{T - T_0}{(lLg/k_s)}, \quad \gamma = \frac{v}{u}, \quad Pe = \frac{uL}{\alpha}, \quad x = \hat{x}/L \quad \text{and} \quad y = \hat{y}/L. \tag{6a–e}$$

2.3. *Interfacial conditions*

The conduction–convection problem defined in equations (3) and (5) are coupled at the wall–fluid interface by the requirement of continuity of temperature and heat flux along $y = 0$. Thus

$$\psi = T \quad \text{and} \quad -k_s \frac{\partial \psi}{\partial \hat{y}} = k_f \frac{\partial T}{\partial \hat{y}}$$

$$\text{and} \quad \hat{y} = 0, \quad 0 < \hat{x} < L \qquad (7a, b)$$

or in dimensionless form

$$\theta = t \equiv f(x) \qquad (8a)$$

and

$$k^* \frac{\partial \theta}{\partial y^*} = \frac{\partial t}{\partial y} \quad \text{at} \quad y = y^* = 0, \quad 0 < x < 1 \qquad (8b)$$

where

$$k^* = \frac{k_s}{k_f}. \qquad (8c)$$

The function $f(x)$, introduced here for convenience in the subsequent analysis, represents the unknown dimensionless temperature along the wall–fluid interface.

The systems defined by equations (3) and (5), coupled through the interface condition given by equation (8), constitute a conjugate heat transfer problem which is solved analytically.

3. PROBLEM SOLUTION

Using the integral transform technique, the solution of the conduction problem defined by equation (3) is readily obtained as [8] :

$$\theta(x, y^*) = \sum_{m=1}^{\infty} \frac{X_m(x)}{N_m(\beta_m)} \cdot \frac{\cosh(\beta_m(A - y^*))}{\cosh(\beta_m A)} \times \left[\int_0^1 X(\beta_m, x') f(x')\, dx' - \frac{I_1(m)}{\beta_m^2 A} \right] + \sum_{m=1}^{\infty} \frac{A}{N_m} \frac{X_m(x)}{\beta_m^2} \cdot I_1(m) \qquad (9a)$$

where $X_m(x)$, β_m and N_m are, respectively, the eigenfunctions, eigenvalues and normalization integrals of the associated eigenvalue problem. The integral $I_1(m)$ is defined by

$$I_1(m) = \int_0^1 X_m(x')\, dx'. \qquad (9b)$$

Using the transformation [8]

$$t(x, y) = w(x, y) \exp\left[\frac{\gamma\, Pe}{2} y - \frac{\gamma^2\, Pe}{4} x\right] \qquad (10)$$

the energy equation (5a) for forced convection is transformed to

$$\frac{\partial w}{\partial x} - \frac{1}{Pe} \frac{\partial^2 w}{\partial y^2} = 0, \quad x > 0, \quad y > 0 \qquad (11a)$$

and the boundary conditions (5b), (5c) and the interface condition (8a) are, respectively, transformed to

$$w(x, y) = 0 \quad \text{at} \quad x = 0 \qquad (11b)$$

$$w(x, y) \to 0 \quad \text{as} \quad y \to \infty \qquad (11c)$$

and

$$w(x, y) = f(x) \exp\left[\frac{\gamma^2\, Pe\, x}{4}\right] \quad \text{at} \quad y = 0. \qquad (11d)$$

The forced convection problem defined by equation (11) is solved by the application of Duhamel's theorem to yield

$$t(x, y) = \frac{2}{\sqrt{\pi}} \exp\left[\frac{\gamma y\, Pe}{2}\right] \int_{y/\sqrt{(4x/Pe)}}^{\infty} \left\{ f\left(x - \frac{y^2\, Pe}{4\eta^2}\right) \times \exp\left[-\eta^2 - \frac{\gamma^2 y^2\, Pe^2}{16\eta^2}\right] \right\} d\eta \qquad (12)$$

where η is an integration variable.

The solutions given by equation (9) for the conduction problem and equation (12) for the convection problem are coupled through the unknown interfacial temperature $f(x)$. The application of the interface condition

$$\left.\frac{\partial t}{\partial y}\right|_{y=0} = k^* \left.\frac{\partial \theta}{\partial y^*}\right|_{y^*=0} \qquad (13)$$

yields the following Fredholm type integral equation for the determination of $f(x)$:

$$\gamma \frac{Pe}{2} f(x) = -k^* \sum_{m=1}^{\infty} \frac{\beta_m}{N_m} \tanh(\beta_m A) \cdot X_m(x) \left[\int_0^1 X_m(x') f(x')\, dx' - \frac{I_1(m)}{\beta_m^2 A} \right] \qquad (14)$$

where $X_m(x)$ are the eigenfunctions, β_m are the eigenvalues, and N_m are the normalization integrals associated with the eigenvalue problem considered. Here we use the Pincherle–Goursat technique to solve this Fredholm type integral equation as outlined on pp. 55–64 of Tricomi [9].

We let

$$C_j = \int_0^1 X_j(x') f(x')\, dx'. \qquad (15a)$$

For the problem considered here we have

$$X_m(x) = \sin(\beta_m x) \qquad (15b)$$

$$\frac{1}{N} = 2 \qquad (15c)$$

and

$$\beta_m = \begin{cases} \dfrac{m\pi}{2} & \text{for} \quad \dfrac{\partial\theta}{\partial x} = 0 \quad \text{at} \quad x = 1; \\ m = 1, 3, 5, \ldots & \\ m\pi & \text{for} \quad \theta = 0 \quad \text{at} \quad x = 1; \\ m = 1, 2, 3, \ldots & \end{cases} \tag{15d, e}$$

Inserting equation (14) into (15a) yields

$$C_j = \frac{2k^*}{\gamma\, Pe} \sum_m^{\infty} \frac{\beta_m}{N} \tanh(\beta_m A) \left[\frac{I_1(m)}{\beta_m^2 A} - C_m \right] \cdot I_2(m, j) \tag{16a}$$

where the integral $I_2(m, j)$ is defined by

$$I_2(m, j) = \int_0^1 X_j(x') X_m(x')\, dx' = \begin{cases} 0 & m \neq j \\ \frac{1}{2} & m = j \end{cases} \tag{16b}$$

and the integral $I_1(m)$, defined by equation (9b), takes the form

$$I_1(m) = \int_0^1 X_m(x')\, dx' = \frac{1 - \cos(\beta_m)}{\beta_m}. \tag{16c}$$

Relation (16a) must hold for each value of m. Then, we have to solve the problem

$$\sum_{m=1}^{\infty} \delta_{jm} C_m + \sum_{m=1}^{\infty} A_{jm} C_m = \sum_{m=1}^{\infty} D_{jm} \tag{17a}$$

where

$$A_{jm} = \frac{2}{\gamma\, Pe} \frac{k^*}{N} \beta_m \tanh(\beta_m A) \cdot I_2(m, j) \tag{17b}$$

$$D_{jm} = \frac{2}{\gamma\, Pe} \frac{k^*}{N} \beta_m \tanh(\beta_m A) \cdot \frac{I_1(m)}{\beta_m^2} \cdot I_2(m, j) \tag{17c}$$

and

$$\delta_{jm} = \begin{cases} 0 & \text{for} \quad m \neq j \\ 1 & \text{for} \quad m = j. \end{cases} \tag{17d}$$

Invoking the orthogonality property of the integral $I_2(m, j)$, equation (17) is reduced to a set of decoupled algebraic equations

$$(A_{mm} + 1) C_m = D_{mm} \tag{17e}$$

which yields the desired coefficients

$$C_m = \frac{2\beta_m \tanh(\beta_m A) \dfrac{I_1(m)}{\beta_m^2 A}}{2\beta_m \tanh(\beta_m A) + \dfrac{Pe\,\gamma}{k^*}}. \tag{18}$$

The block temperature given by equation (9a) can be rewritten in the form

$$\theta(x, y^*) = 2 \sum_{m=1}^{\infty} \sin(\beta_m x) \frac{\cosh(\beta_m (A - y^*))}{\cosh(\beta_m A)} \cdot \left[C_m - \frac{I_1(m)}{A\beta_m^2} \right] + 2 \sum_{m=1}^{\infty} \sin(\beta_m x) \cdot \frac{I_1(m)}{A\beta_m^2} \tag{19}$$

whereas the block–fluid interface temperature becomes

$$f(x) = 2 \sum_{m=1}^{\infty} C_m \sin(\beta_m x) \tag{20}$$

and the fluid temperature takes on the form

$$t(x, y) = \frac{4}{\sqrt{\pi}} \exp\left(\frac{\gamma y\, Pe}{2}\right) \int_{y\sqrt{(4x/Pe)}}^{\infty} \left\{ \exp\left(-\eta^2 - \frac{\gamma^2 y^2 Pe^2}{16\eta^2} \right) \cdot \sum_{m=1}^{\infty} C_m \sin\left[\beta_m \left(x - \frac{y^2 Pe}{4\eta^2} \right) \right] \right\} d\eta. \tag{21}$$

Once the coefficients C_m are known from equation (18), various quantities of practical interest can be determined from their definitions. For example, the dimensionless local heat flux at the interface is given by

$$q_w(x) = \frac{\gamma\, Pe}{2k^*} f(x) = \frac{\gamma\, Pe}{k^*} \sum_{m=1}^{\infty} C_m \sin(\beta_m x) \tag{22}$$

where

$$q_w(x) = \frac{\hat{q}_w(\hat{x})}{gl}$$

with $\hat{q}_w(\hat{x})$ being the heat transfer rate across the interface. The average heat flux can be computed as

$$\bar{q}_w = \int_0^1 q_w(x)\, dx = \frac{\gamma\, Pe}{k^*} \sum_m C_m I_1(m). \tag{23}$$

The local Nusselt number is given by

$$Nu = \tfrac{1}{2}\gamma\, Pe. \tag{24}$$

In order to obtain an estimate for the thermal penetration depth, we let

$$t|_{y \to \delta_{th}} \to 0.005 \tag{25}$$

which is used to estimate $\delta_{th} = \hat{\delta}_{th}/L$. This is a suitable parameter for determining the appropriate cooling channel width formed by electronic circuit boards for equipment; or spacings between hot blocks of metal, ceramic, or glass to be cooled on conveyor belts.

4. RESULTS AND DISCUSSION

The foregoing analytic solutions are used to examine the effects of various parameters such as the Peclet number, velocity ratio, thermal wall conditions and the thermal conductivity ratio on the block surface temperature, surface heat flux and the penetration depth. Thus, the parameters Pe, v/u and k_f/k_s are lumped into a dimensionless group, $Pe\,\gamma/k^*$; the thermal side wall conditions are prescribed as $\theta = 0$ at

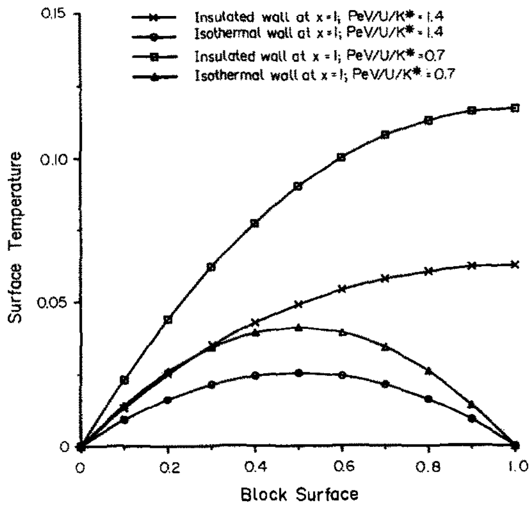

FIG. 2. Interface temperature distributions for different thermal side wall conditions and lumped system parameters.

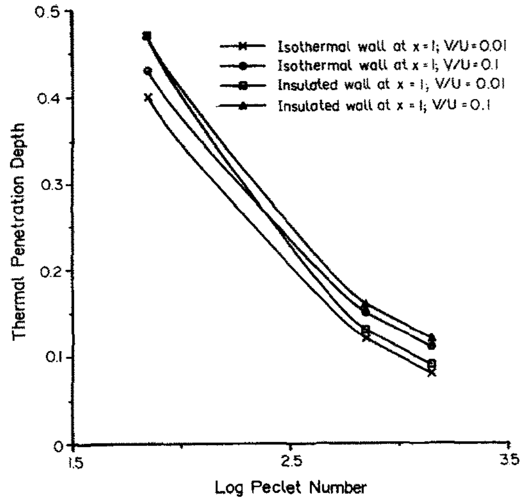

FIG. 3(b). Thermal penetration depths vs Peclet number for different velocity ratios and thermal side wall conditions ($k_s/k_f = 10$).

$x = 0$ and $\theta = 0$ or $\partial\theta/\partial x = 0$ at $x = 1$. The results are summarized in Figs. 2–5(b).

Figure 2 depicts various interfacial or block surface temperature profiles for different group numbers and thermal exit wall conditions. The surface temperature increases as $(Pr\,\gamma)/k^*$ decreases, because less heat is carried away when Pe, γ or k_f are lowered or k_s is increased.

The maximum thermal boundary layer thickness or thermal penetration depth is of interest for design purposes. Figures 3(a) and (b) show $\delta_{th} = \tilde{\delta}_{th}/L$ as a function of $\log(Pe)$ for different $\gamma = v/u$ and $k^* = k_s/k_f$ and with constant side wall temperature ($\theta = 0$) or constant heat flux condition ($\partial\theta/\partial x = 0$). As expected, the thermal penetration depth decreases rapidly with higher Peclet numbers, $Pe = Re\,Pr$, because the boundary layer becomes thinner with larger Reynolds numbers. Both a stronger transverse velocity component and the insulated end wall condition generate higher penetration depths; however, the type of thermal wall condition has only a marginal effect on δ_{th} (cf. Fig. 3(a)). A higher k^*-value shifts the δ_{th}–$\log(Pe)$ curves disproportionally upwards, reflecting the nonlinear coupling effect between the fluid conductivity and the vertical velocity component (cf. Fig. 3(b)).

Figures 4(a) and (b) show the surface average heat flux, $\bar{q}_w$, as a function of $\log(Pe)$ for different velocity ratios, thermal boundary conditions and conductivity ratios. The adiabatic exit wall condition causes higher block surface heat fluxes because of the higher interfacial temperatures (cf. Fig. 2). The mean surface heat

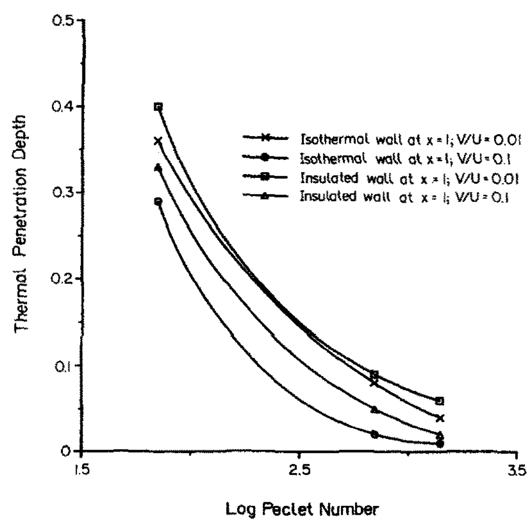

FIG. 3(a). Thermal penetration depths vs Peclet number for different velocity ratios and thermal side wall conditions ($k_s/k_f = 1.0$).

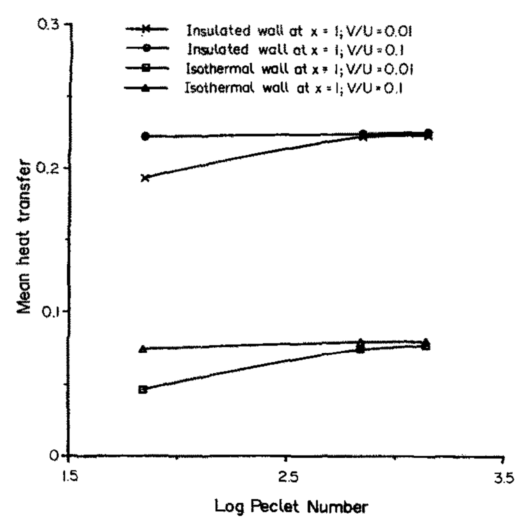

FIG. 4(a). Average interface heat flux vs Peclet number for different velocity ratios and thermal side wall conditions ($k_s/k_f = 1.0$).

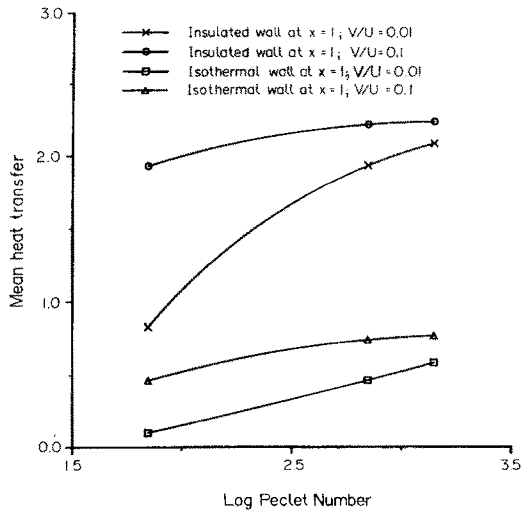

FIG. 4(b). Average interface heat flux vs Peclet number for different velocity ratios and thermal side wall conditions ($k_s k_f = 10$).

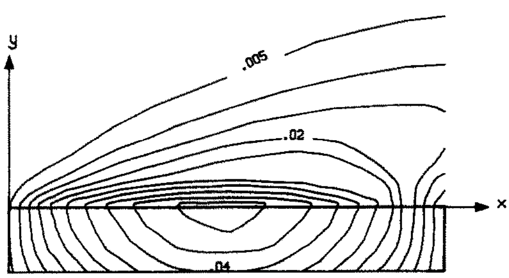

FIG. 5(a). Block and fluid temperature contours for isothermal surface conditions at $x = 0$ and 1.

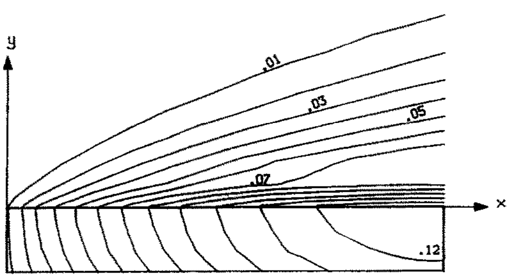

FIG. 5(b). Block and fluid temperature contours for isothermal surface conditions at $x = 0$ and insulated wall condition at $x = 1$.

flux increases significantly with increasing conductivity ratios (cf. Figs. 4(a) and (b)), and at larger k^*-values the effects of the Peclet number and velocity ratio are more pronounced.

Temperature contours for the case of isothermal block surfaces are shown in Fig. 5(a). The block isotherms are slightly asymmetric because of the mild cooling effect of the convective stream. In contrast, Fig. 5(b) depicts the effect of one-sided wall insulation, a possible worst-case scenario for cooling of multiple, closely-spaced blocks [10]. In this case the block and fluid temperatures are higher than in Fig. 5(a), reaching a maximum at the end of the block ($x = 1.0$). As a result, the thermal penetration depth for the insulated block is greater ($\delta = 0.35$) than under the isothermal wall condition ($\delta = 0.25$).

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SOLUTION ANALYTIQUE DU PROBLEME DE TRANSFERT THERMIQUE CONJUGUE POUR UN ECOULEMENT AUTOUR D'UN BLOC CHAUFFE

Résumé—Une solution analytique est développée pour le transfert thermique laminaire conjugué pour un bloc rectangulaire ayant une source interne constante et étant soumis à la convection forcée sur le sommet, à différentes conditions thermiques sur les cotés et à une condition de paroi adiabatique sur la base. La solution du problème idéalisé est intéressante pour l'analyse du refroidissement d'un bloc unique. Les résultats numériques présentés illustrent les effets du nombre de Péclet, du rapport des conductivités et du type de conditions thermiques aux limites sur les profils de température et les distributions du flux thermique le long de la surface supérieure du bloc.

ANALYTISCHE LÖSUNG FÜR DAS PROBLEM DES KONJUGIERTEN WÄRMEÜBERGANGS AN DER RÜCKSEITE EINES UMSTRÖMTEN BEHEIZTEN BLOCKES

Zusammenfassung—Für den stationären laminaren zweidimensionalen konjugierten Wärmeübergang aus einem rechteckigen Block mit konstanter innerer Wärmequelle wird eine analytische Lösung entwickelt. Die Oberseite des Blockes ist einer erzwungenen Anströmung ausgesetzt, an den Seiten herrschen unterschiedliche thermische Randbedingungen, und die Unterseite ist adiabat. Die Lösung dieses idealisierten Problems ist für die grundlegende Untersuchung der Kühlung eines Einzelblockes von Bedeutung. Die vorgelegten Ergebnisse numerischer Berechnungen zeigen den Einfluß der Peclet-Zahl, des Verhältnisses der Wärmeleitfähigkeiten sowie der Art der thermischen Randbedingung auf die Temperaturprofile und die Verteilungen der Wärmestromdichte an der oberen Deckfläche des Blockes.

АНАЛИТИЧЕСКОЕ РЕШЕНИЕ СОПРЯЖЕННОЙ ЗАДАЧИ ТЕПЛОПЕРЕНОСА ПРИ ОБТЕКАНИИ НАГРЕТОГО БЛОКА

Аннотация—Получено аналитическое решение задачи стационарного двумерного взаимосвязанного теплопереноса для ламинарного течения в прямоугольном блоке с постоянным внутренним источником в случае вынужденной конвекции у верхней поверхности, когда на боковых поверхностях выполняются различные тепловые граничные условия, а основание является адиабатическим. Идеализированное решение задачи представляет интерес для анализа охлаждения единичного блока. Полученные численные результаты иллюстрируют влияние числа Пекле, отношяние теплопроводнстей и типа тепловых граничных условий на профили температур и распределения тепловых потоков на верхней поверхности блока.